Kiln Planning, A Cutting Stock approach

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Abstract:

Cutting stock problem deals with selection of best layout from a finite number n of demand items and a theoretically infinite set of base materials or stocks. This article describes a layout generation algorithm to find layouts for cutting all demands of circular items with diameters of $\{d_1, d_2, d_3, ...\}$ *out of the base rectangle so that the minimum number of base rectangles is required. The layout and total number of layouts that are used is as a function of demand for a specific order. A generalized problem for cases with demand for circular items of different size* $\{r_1, r_2, r_3, ...\}$ *with a finite number of base rectangles plates is also discussed.*

Keywords: cutting stock problem, Pattern generation, Integer programming, Kiln planning,

Introduction

Cutting and packing Problem belong to an old and very well known family , called CSP in [7,8,9]. A great number of problems are essentially based on the same logical structure of the *Cutting* and *Packing* problems. These include: Cutting Stock and Trim Loss; Template Layout; Coil Slitting; Placement; Packing and Depletion; Bin Packing; Strip Packing; Vector Packing; Knapsack Problems; Vehicles Loading; Pallet Loading; Container Loading; Cargo loading; Assortment; Nesting; Partitioning Problems; Capital Budgeting And Change Making; Memory Allocation; And Multiprocessor Scheduling Problems.

The first documented form for this type of problems appeared in the *paper Industry* with the first known formulation stated, in 1939, by the Russian economist Kantorovich. Gilmore and Gomory introduced, in 1961, a delayed pattern generation technique for solving a one-dimensional cutting problem using linear programming [3].

Therefore, CSP is a family of natural combininatorial optimization problems, admitted in nemerous real world application from computer science,Industrial Engineering , Manufacturing , Production process , etc. Concerning the tools & techniques to solve this kind of problems in industry there has been many researches on rectangular based CP[1,2,5,6,7,11].

In an attempt to solve the different variations of the cutting and packing problem, more than 800 papers have been published. The solution approaches include: [12]

- Linear programming (LP) approaches.
- Dynamic programming approaches.

- Heuristic based approaches including best first search, simulated annealing and some new approaches.
- Genetic algorithm (GA) approaches.
- Expert Systems (ES) approach.

The large number of publications in this area is not surprising because of two realities;

1. The Cutting Stock problems are classified as NP hard, which means it is difficult to get optimal Solutions mathematically, and
2. There is no common approach to solve different configuration (classes) of the problem. Any change in the problem configuration will affect the efficiency of the solution used.

The overall architecture of solving an applied CSP problem can be defined as a three level Tools & techniques including 1)pattern generation , 2) linear programming for pattern selection and 3) Tools for implementation . [12]

In SCP, since various heuristics have been proposed with a varying degree of performance under different problem domains to select the best suitable pattern according to a specific objective, there is a real need for an efficient knowledge base. Literature review on CSP shows that there have been many researches on pattern selection and especially for rectangular objects. This paper focuses on pattern generation and especially for a group of problems dealing with circular objects. These kinds of objects are mostly used in ceramic industry and porcelain plants where there is lack of optimization techniques to improve production facility utilization.

Problem Definition

This paper relates to the problem of kiln scheduling in ceramic industry which can be treated as a CSP. For a clear understanding of the problem, Imagine that you work in a porclain manufacturing company, and you are a manager in the kiln charging & planning division. You have a number of circular tablewares such as plates,dishes with different width waiting to be set on kiln wagon. yet different customers want different numbers of tablewares of various-sized widths. How are you going to plan the kiln so that the least amount of left-over sapces are wasted? Or in other means to find a layout mix to make use of least amount of space . This turns out to be an optimization problem, or more specifically, a combinition of pattern generation and an integer linear programming problem.

We can consider the problem to be an integer linear program where the variables x are the number of each pattern to be selected. The objective function is to minimize the number of kiln wagons(or maximize the capacity of kiln) . The constraints in the problem require that we select enough parts with certain patterns to fulfill the orders that we have received. In our example, the objective and the constraints can easily be formulated as mathematical equations.

The problem is reported in many cases in stamping & blanking of round products in metal forming industry where the flexibility of machines is good enough to handle different orders of different diameters. The application of the problem formulation can also be used in glass industry, packing industry

The cutting stock problem arises from many physical applications in industry. Clasic type problems deal with rectangles or traingles (which can be combined to rectangles). The problems is solved through a linear programming approach. In this type of problems the patterns are introduced during optimization.
In real type problems there are a lot of applications where circular patterns should be selected (packed) from a base material(in a packing). But as the circular patterns is concerned there is not a basic theorem to be used for generating patterns.
The basic conditions of the model are as follows:

1. A wagon car with a rectanglur area is used for setting the circular parts
2. There are Circular parts (saggars[1]) with different radius that should be settled on the wagon car
3. The objective is to minimize the waste (free area) or maximize the capacity of a wagon

Generation of different pattern with unspecified layout isnot a practical situation in industrial environment.therefore, a simplified and practical set of rules are established. The rules are:

- The parts in a row are the same.
- The parts are layed out in a row parallel to length or width of the wagon car
- The center of a parts in a row are on a stright line

Pattern generation

The problem of finding optimum number of patterns to maximize the objectives (benefit,customer satisfaction,…)of any company is a combination of a integer programming and a pattern generation algorithm.
Based on assumptions made , there should be patterns including only one item on a kiln car. This basic idea will make the problem more convienient for parts setting and a general theorem for other type of patterns is developed.

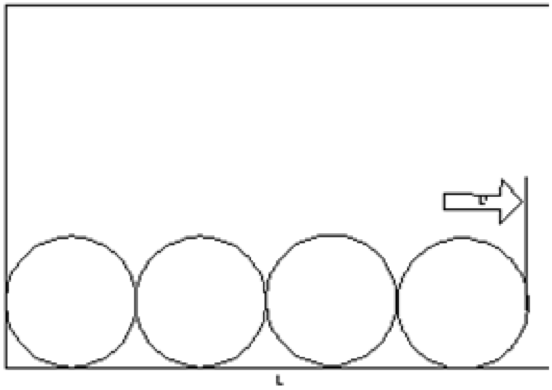
Single type circular parts pattern

Consider there is a flat bed with a length (width) of L and an item with a diagonal of D. The problem of finding the suitable patterns will be three cases:

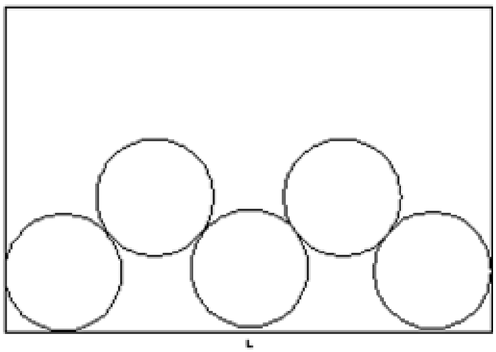
- The length (width) will be filled up with an integer no of circles.(there is no free length)
- The length(width) is grater than intger no of circles.(there is a free length

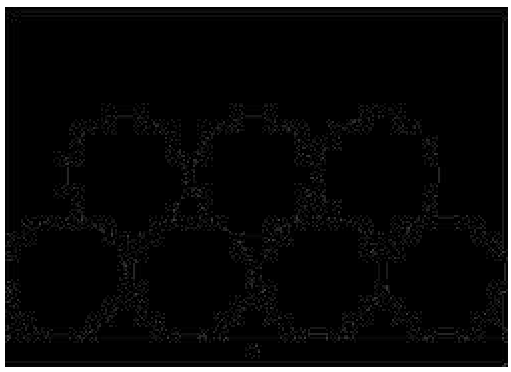
The second case should be considered for finding a best alternative to make use of free area due to unused length of bed. investigation of this type will introduce two alternatives which is shown in figure(1).

[1] A saggar is a container for pilling up parts in a kiln in order to prevent Joining of parts.



Possible layout - Type P





Type A Type B

figure 1- Alternate models for the single item pattern

A possible layout for single item is shown in figure 1. In this layou a gap between the edge of the bed with the last item(i.e. nth item) .we call this gap L'. The area which is filled by the items and cannot be used is estimated with a recatangluar block of the size L*D.

To make use of the gap and utilization of area,there is two choice. The first one which is shown in pattern "Type A", can be designed by adding only one item to the base design.(i.e. n+1 item) . The area of a minimum rectanle block will be changed due to changes in L' (i.e. F(x)=f(L,D,L')).

As it is shown ,Pattern type B will be designed by distrubting of base row to it's maximum length and adding (n-1) item.

To describe utiliozation of all possible layouts of a minimum block rectangle, a function is used. Ratio of Such a function f(D,L) with it's base Type P pallete can be calculated as a geometrical function of the main parameters. This ratio R can be used for generation criteria of new layout.

Theorem 1 : In type A layout where the ratio R is $L' \phi \; d(\sqrt{n^2-1}+1-n$*, Layout of type A is better than P type.*

Proof: consider we call the filled area by $S_1 \& S_2$ for type P & A layouts.Also, consider that the area of the item with Diameter of d is called A. then we can write :

$y_{block\_\max} = y + d$

$$S_2 = \frac{(n+1)A}{L(y+d)} \qquad S_1 = \frac{nA}{Ld}$$

where
y :is the distance between centers of two adjacent items in vertical direction
&
x : is the distance between centers of two adjacent items in horizontal direction

According to definition, the ratio R can be calculated as:

$$R = \frac{S_2}{S_1} = \frac{\frac{(n+1)A}{L(L+d)}}{\frac{nA}{Ld}} = \frac{(n+1)d}{(y+d)n}$$

the criteria of selecting a layout is to find a value for y where one of layouts is better than the others. Therefore we can write :

$$R = \frac{S_2}{S_1} \phi 1 \Rightarrow \frac{n+1}{n}.\frac{d}{y+d} \phi 1 \Rightarrow y \ \pi \ \frac{d}{n}$$

From the geometry of circles , the condition for the x axis can be calculated as :

$$y^2 = d^2 - x^2 \Rightarrow y = \sqrt{d^2 - x^2} \Rightarrow \sqrt{d^2 - x^2} \ \pi \ \frac{d}{n}$$

$$x \ \phi \ d\sqrt{1 - \frac{1}{n^2}} \qquad \text{or} \qquad x \ \phi \ \frac{d}{n}\sqrt{n^2 - 1}$$

from the other point of view we have a relationship between the number of items and the length of layout which leads to following equations:

$$nx + d = L \rightarrow x = \frac{L-d}{n} \Rightarrow L > d(\sqrt{n^2 - 1} + 1)$$

note: the x distance is biger than the diameter (i.e. two times the radious of the adjacent circles) of an item because there is a free space between the items.

Also there is another equation for type P layout which leads to following equations:[2]

$$nx + L' = L \rightarrow nd + L' > d(\sqrt{n^2 - 1} + 1)$$

$$\boxed{L' > d(\sqrt{n^2 - 1} + 1 - n)}$$

Theorem 2: In type B layout where the ratio R is $L' \phi \ (n-1)d(\frac{2}{n}\sqrt{2n-1} - 1)$*, Layout of type B is better than P type.*

Proof: consider we call the filled area by S_1 & S_2 for type P & B layouts.Also, consider that the area of the item with Diameter of d is called A. then we can write :

$$y_{block\_max} = y + d$$

[2]From the equations the location of the items center point are :

$$x = \frac{L-d}{n} \quad \& \quad y_{first\_row} = \frac{d}{2} \quad \& \quad y_{second\_row} = \frac{1}{n}\sqrt{n^2 d^2 - (L-d)^2}$$

$$S_2 = \frac{(2n-1)A}{L(y+d)} \qquad\qquad S_1 = \frac{nA}{Ld}$$

where
y :is the distance between centers of two adjacent items in vertical direction
&
x : is the distance between centers of two adjacent items in horizontal direction

According to definition, the ratio R can be calculated as:

$$R = \frac{S_2}{S_1} = \frac{\dfrac{(2n-1)A}{L(y+d)}}{\dfrac{nA}{Ld}} = \frac{(2n-1)d}{(y+d)n}$$

The criteria of selecting a layout is to find a value for y where one of layouts is better than the others. Therefore we can write :

$$R = \frac{S_2}{S_1} \phi\, 1 \Rightarrow \frac{2n-1}{n}.\frac{d}{y+d} \phi\, 1 \Rightarrow y \,\pi\, \frac{d(n-1)}{n}$$

From the geometry of circles , the condition for the x axis can be calculated as :

$$y^2 = d^2 - x^2 \Rightarrow y = \sqrt{d^2 - x^2} \Rightarrow \sqrt{d^2 - x^2} \,\pi\, \frac{d(n-1)}{n}$$

Then $\qquad x \,\phi\, d\sqrt{2n-1}$

from the other point of view we have a relationship between the number of items and the length of layout which leads to following equations:

$$(n-1)(2x) + d = L \rightarrow x = \frac{(L-d)}{2(n-1)} \qquad\qquad \Rightarrow L > d(1 + \frac{2(n-1)}{n}\sqrt{2n-1} - 1)$$

note: the x distance is biger than the diameter (i.e. two times the radious of the adjacent circles) of an item because there is a free space between the items.

Also there is another equation for type P layout which leads to following equations:[3]

$$nx + L' = L$$

$$L' > d(n-1)(\frac{2}{n}\sqrt{2n-1} - 1)$$

<u>Criteria for pattern selection</u>

As a result of the equations exist in selected patterns, there is a relationship between R & n shown in figure (2).

[3]From the equations the location of the items center point are :

$$x = \frac{L-d}{n}.(n-1) \quad \& \quad y_{first\_row} = \frac{d}{2} \quad \& \quad y_{second\_row} = \frac{1}{2(n-1)}\sqrt{(2d(n-1))^2 - (L-d)^2}$$

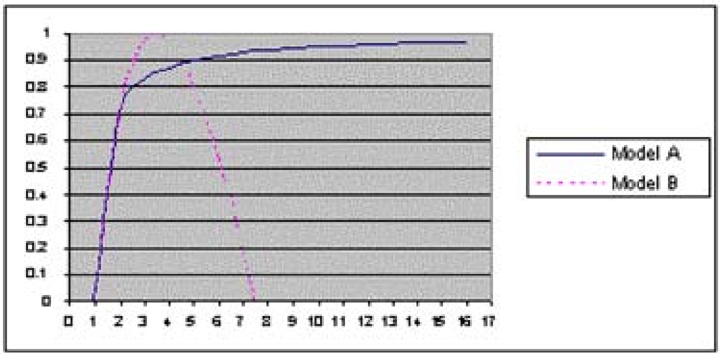


Figure (2).graph of relationship between R ratio and n

based on the graph there is a general criteria which can be used in a pattern generation algorithms:

Lemma: *"for n>4 , B type patterns is better than A type & for the cases of* $n \leq 4$ *pattern A is better than B type ".*

The above lemma is a useful criteria where Extension of a pattern is just copying the minimum block. A further investigation shows that there would be an oppurtunity for improvemen due to similarity of patterns .This improvement is a fact of using some of free spaces in the base block for the extension block which makes the total area usage less than two times a block area.theorem 3 shows the condition and values of improvement.

Theorem 3: In every layout there is an improvement index for extension of minimum block which is a function of L ratio for every n.

Proof: consider we want to find feasible location of the next block.It is obvious that the center of the next block can be on a line of $y \geq y_{Block-\max} + d/2$ .From the geometry of the blocks it can be shown that there is a minimum location which makes improvement of area usage up to 7.2% for B type layout and 32% for A type.

(a)-feasible crossing *(b)-infeasible crossing*

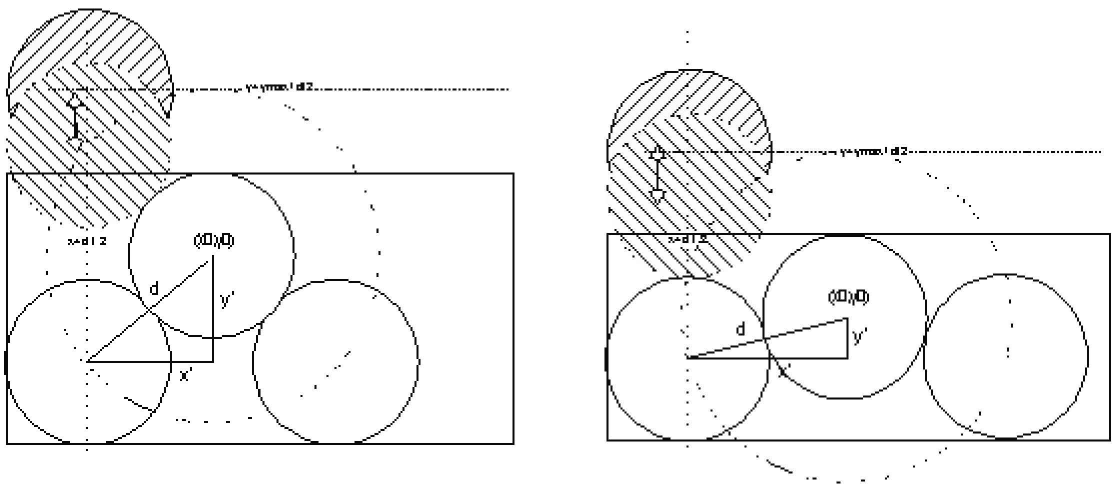


Figure (3).graph of relative position of items in extended area

From geometry and refer to figure (3) we have $y'=\sqrt{d^2-x'^2}$ where

$$x'=\frac{2(L-d)}{n} \quad \text{for type B pattern}$$

$$x'=\frac{(L-d)}{n} \quad \text{for type A pattern}$$

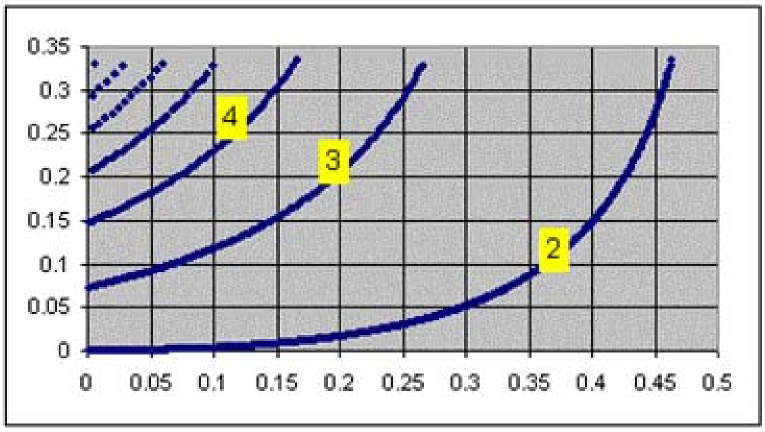
The minimum location for the next items in extension direction will be on a circle with radious of d and center of (x_0, y_0). It can be shown that cross of this circle and the line x=r (i.e. the location of the center of extended circle) will lead to a optimum y value (least location) as below:[4]

$$y = d/2 + 2\sqrt{d^2 - x'^2} \quad \text{if} \quad y \geq 3(d/2)$$

$$y = 3(d/2) \quad \text{otherwise}$$

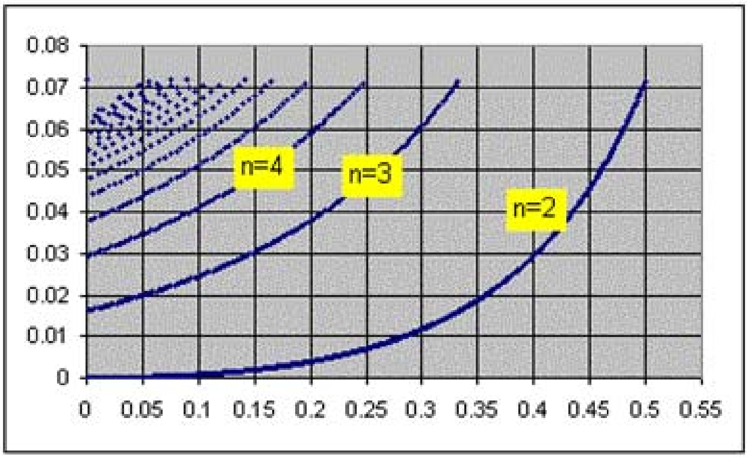
figure (2) shows a sample of the feasible condition and figure b shows an infesible crossing where the $y = 3(d/2)$ is the optimum solution.

Improvement index is based on the difference between $\{(y_{Block\_max} + d/2) - y_{opt}\}$ which leads to an improvement of area. It can be shown that the Improvement index will increase where the L ratio(i.e. L'/L) incresaes. Index improvement curves for A&B type layouts is shown in figure (4) .



(a)- improvement index curves for type A layout

[4] $(y-y_0)^2 = d^2 - (x-x_0)^2, x = x_0 = d/2$



(b)- improvement index curves for type B layout
Figure (4).graph of improvement index in relation to L ratio for diferrent n

Two type circular parts pattern

Consider there is a flat bed with a length (width) of L and two items with diagonals of D & D'. The problem of finding the suitable patterns will be three cases:

- A layout type P is chosen for the first item and the items are distributed horizontally (vertically) . the second item will be added by the priciple of layout type B.
- A minimum block for each item is calculated . the rectangular blocks are added to each other.

The second case should be considered for finding a best alternative to make use of free area due to unused length of base bed. investigation of this type will introduce two alternatives which is shown in figure(5).

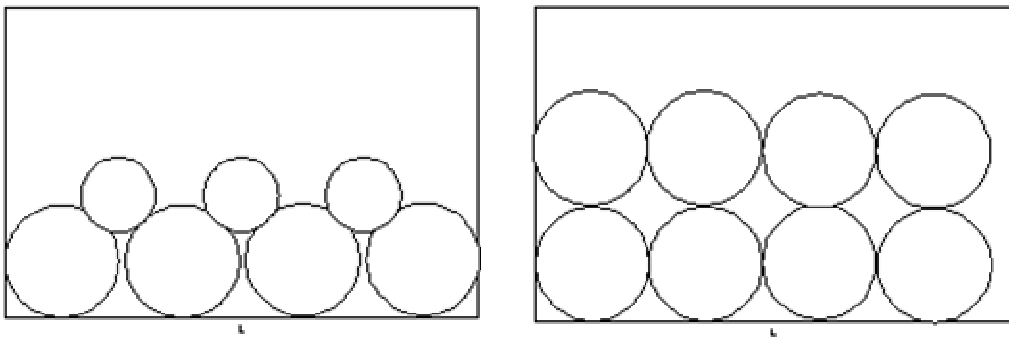


figure 5- Alternate models for the single item pattern

To evalute the location of second row circular parts , you should draw a circle with a radius with a length equal to total radius of both circles. Therefore , the second row's center of parts are on a curve which is at a same distance from first row. The results of cross section of two curves (each from a adjusent parts) are shown as a point .(let's call it "m". these points are the centers of next level parts. A block containing two rows of circular parts are chosen as a base rectangle. Adding these base rectangles can genertae a specific pattern.

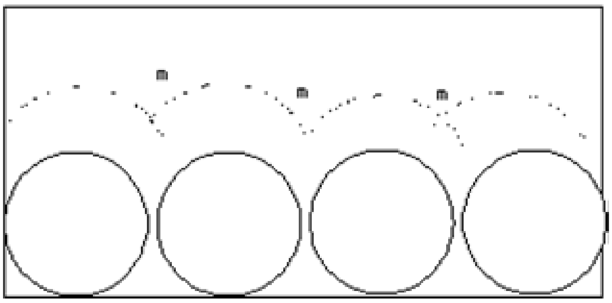


figure 6- center points of second row parts

Modeling & optimization

if a Block rectangle has been computed which contains a set of demand circles and if the dimensions of the stock circles are known, the maximal needed pattern can be found subject to the given number of the stock circles.

Now we have a linear programming problem to solve. There is one problem to our formulation so far. We first created all possible patterns that might be created from a base bed. Given these patterns we form the linear programming relaxation of the problem. If there are many different orders , it is possible for there to be an exponential number of patterns that need to be included in the linear program (LP). As a result, the LP will be too large to even formulate and impossible to solve.[3]

For each pattern j, let A_{ij} be the number of pieces of each items. If we let x_j be the number of times the jth pattern will be used, then $A_{ij}*x_j$ must be greater than or equal to the required number of pieces for the jth product. This is the constraint. What we want to maximize(minimize) is the number of products (free space, which is the sum of all C_jx_j). Another important constraint is that all x_j have to be non-negative. This seems obvious to us, for there can never be a negative number of patterns.
Since integer linear programs are difficult to solve, we will relax the problem to first consider a fractional solution. Once we have obtain the fractional solution, we can apply an integer programming algorithm to remove the fractional portion. [4]

Application: Kiln scheduling

A reasonable question to ask is, do we really have to generate all the possible patterns even before solving the problem? The answer is no. The delayed column generation method solves the cutting stock problem by starting with just a few patterns. It generates additional patterns when they are needed. The new patterns are introduced by solving another optimization problem called the knapsack problem. The knapsack problem has well-known methods to solve it, among which are branch-and-bound and dynamic programming. The delayed column generation method turns out to be much more efficient than the original approach.

In Another approach, We choose an initial set of patterns to include in the model and solve the linear program. Since it is likely that we chose the right set of patterns if we introduce utility ratio(utility ratio=filled up space / total space), we used the critria of 70% or more utility factor. We generate total patterns and used a linear programming software to solve the problem.

Modeling Optimum

This problem can be formulated as a LP-problem as follows:

Minimize $\sum_{k} C_{K} X_{K}$

Subject to

$$\sum_{K} X_{K} \geq TOTALWAGON$$

$$\sum_{K} X_{K} * S_{K,I} \leq TOTALSAGGAR$$

Where

X_{K} : is the number of pattern k

$S_{K,I}$: is the quantity of ith saggar (or part) in kth pattern

Here TOTALWAGON is the total base place for layout of generated patterns.(in our case it is the number of wagon inside the kiln and waiting in the queue outside the kiln). In case of total demand of products,variable TOTALSAGGAR stands as a boundry to the solution.

Computational Results

The problem in real cases has been tested with several instances taken from real demands of a porclain factory, containing between 12 and 36 circular sahpes. A basica programm develpoed for the case of pattern generation and acceptance criteria of 50% & more. A PC-486 (33 MHz) was used to measure the time requirements of the algorithm. The time in all cases are less than 5 seconds for problems up to 36 products. The output of the pattern generation algorithm are used to solve the optimization problem. Lindo software are used to solve linear programms. In most cases, the solutions are good enough for a near optimal integer solutions. In case of linear solutions which are not suitable, a branch & bound algorithm for finding integer solutions are introduced. The result of algorithms for pattern generation makes a 15-25% improvement . Applying the LP optimization model makes a total improvement of 25-33% of the solution.

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