

CAPACITY PRICE DECISIONS, A MANUFACTURING YIELD MANAGEMENT PERSPECTIVE

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Abstract

This paper focuses on formulating capacity-price trade off problem in Yield Management for manufacturing industry by drawing motivation from the remarkable success of yield management implementation in airlines. We present a framework for customer segmentation and lead-time demand management to change standard production and capacity planning problem to Yield management problem. For a planning period of T, we formulate the model with the objective of optimizing both price and capacity utilization factor, simultaneously. We develop an innovative two-stage dynamic programming model to help practitioners to using the benefit of a dynamic model with reasonable computational effort. To formulate the problem in a general framework, we devise a demand model with a probability function independent structure. In current practice, there is no alternative and procedure for manufacturer, as well as customers to take advantage of using unfulfilled capacity in discounted offers. We also identify some important challenges and devise a set of rules to assist decision makers in manufacturing. The parameters of the model can be supported by sales and typical production planning data base.

Keywords: Revenue management, Yield management, Capacity planning, Pricing, Order booking, Assemble to order, Make to order, Dynamic programming

1. Introduction

The intense global competition in the 1980s forced world-class organizations to offer low cost, high quality and reliable products with greater design flexibility. Manufacturers planned to utilize just-in-time and other management initiatives to improve manufacturing efficiency and cycle time. Therefore, manufacturing industry has been struggling to remove the obstacles to high machine utilization and maximum production output from serial production lines.

Our approach to improve utilization and production output is to apply yield management concepts. First, we show that there exists a close link between yield management and manufacturing. Then, we develop a mathematical

model to derive some rules for increasing capacity utilization of the system. In this approach, we focus on capacity planning instead of traditional production planning.

Our proposed model takes into account the demand behavior of the market and combines it with production policies and the historical sales pattern as well as consideration of having certain flexibility in product and process. The scope of model is the focus on interrelationship of the market segmentation, market channels and customer decision-making rule to optimize the revenue based on price-capacity tradeoff for different customers and the manufacturer.

The general model of our approach to cover YM discipline in new area of non-service application is depicted in figure 1.

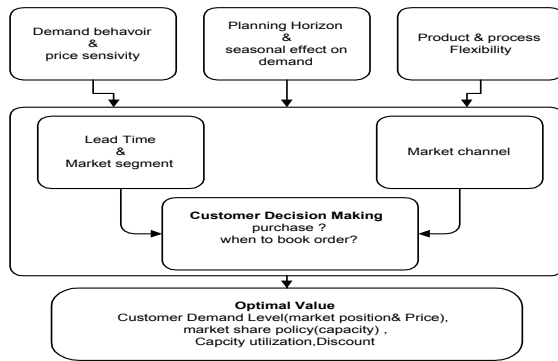


Figure 1 - model framework for manufacturing Yield management

The paper proceeds as follow. Section 2 provides the concepts of manufacturing yield management. In section 3, we develop a mathematical model to represent the problem. A two-stage dynamic programming approach is introduced in section 4. In section 5, we introduce further development of model and managerial

implication on model. Numerical illustrations are presented in section 6 followed by conclusion and further research area in section 7.

2. Literature Review

Even though YM has been widely used in service industry, it needs to be redefined for decision making in manufacturing section. Therefore, we review the literature regarding the market trend and general aspect of revenue management first. For a general literature review in revenue (or yield) management one can refer to Van Ryzin and McGill (1999). The papers address the Yield management business strategy designed to help them optimize their revenue. Broadly defined, Yield/Revenue Management is to sell the right inventory, to the right customer, at the right time, and at the right price. Bitran and Caldentey (2003) make a similar general review on revenue management from price modeling point of view.

Accurate forecasting is extremely important for a model because of its direct impact on revenues. Early work in this area investigated the Poisson, Gamma and Negative Binomial models for final demand (Beckmann ,1958). There are many researches such as (Montgomery, 1976) and (Rajopadhye, 2001) available on forecasting data in business application include regression techniques, time series models and box and Jenkins models. Many researchers modelled the booking requests as stochastic arrival process to determine the total demand distributions especially for airline industry (Ayodhiramanujan, 2003) and (Harris and Pinder, 1995) and

(Weatherford, 2002) and (William, 2002). Several variations for estimating cancellations, no-shows etc. were investigated ((Taylor,1962) and later research showed that the normal distribution is usually a good continuous approximation for aggregate demand (Belobaba ,1987). To conduct more realistically models of the variance of the final demand, other processes such as the stuttering Poisson (Rothstein (1968, 1971)) and the batch Poisson process (Bobkowski (1958)) have been proposed. The forecasting literature is broadly touched from different business point of view but as our model is independent from every stochastic demand, we will not focus on this subject.

The seat-inventory control problem is that of determining how to allocate seats/capacity across multiple fare classes. The earliest seat-inventory control models focused on single product offers, starting from Littlewood's rule for two-fare classes (Bhatia and Parekh ,1973) Much work was done on testing the assumptions under which Littlewood's rule is optimal and on empirical testing of its performance (Richter,1982,MAYER,1976). Belobaba extended Littlewood's rule to multiple fare classes and proposed the Expected Marginal Seat Revenue (EMSR) rule, which is not optimal in general apart from the two-fare case but is very easy to implement and usually gives good results for common demand distributions (McGill, 1989; Curry, 1990; Wollmer, 1992; Brumelle and McGill, 1993). Work has been done on extensions of EMSR to produce better approximations of optimal booking policies (Van Ryzin and McGill, 1998).Work on this subject is too extensive to cite here, see (Van Ryzin and McGill, 1999) for a collection of references. The work which is most related to our work is related to dynamic programming approach. However, dynamic programming is widely used on a different but similar area's in inventory management; we haven't found many works on this subject in Yield Management literature.

pricing is as an important part of the Yield management practice. Work on this can be found in (GALLEGO and VAN RYZIN (1994, 1997)), where dynamic pricing problems are treated to determine optimal pricing policies. Little has been published on joint capacity allocation/pricing and market segmentation. The main focus of our work is on the joint capacity pricing issues, where the effect of remaining seats on pricing should be addressed. Some work can be found in (Botimer (1994)).

As Guerrero (1992) and Whybark and Wijngaard (1994) argue, operational coordination between marketing and production functions is becoming an increasingly important issue. Policy explored at the tactical level to ensure effective management of demand and capacity (Sridharan, 1998), which is a critical issue especially when the goals of both marketing and production Functions are concurrently considered (Crittenden, 1995).

In both manufacturing and service operations management literature, researchers have addressed the issue of allocating scarce resources to competing classes of demand. Several researchers dealing with the MTS manufacturing environment have written about the issue of allocating scarce inventory among competing classes of customers (Topkis, 1968; Tan, 1974; Nahmias and Demmy, 1981; Cohen et al., 1988; Jackson, 1988; Haynsworth and Price, 1989; Ha, 1997; Chun, 2003). In the service operations management area, researchers have dealt with the issue of allocating scarce fixed capacity among competing classes of demand in the context of airlines (Kimes, 1989; Brumelle et al., 1990; Smith et al., 1992), hotels (Relihan, 1989) or rental car agencies (Carrol and Grimes, 1995). The work done in this area is referred to as perishable asset revenue management (PARM) (Weatherford and Bodily, 1992; Harris and Pinder, 1995).

How ever, prior work on applying the PARM concept for short-run capacity management has been limited in MTO manufacturing environment, Bertrand and Sridharan (2001). The critical link between production and marketing functions is investigated by Barut et al. (2003), Guerrero (1992), Whybark and Wijngaard (1994) and Sridharan (1998). There are tremendous literatures on related subject, but the most related papers, even though in concept, are Harris and Pinder (1995) and Chun (2003).

For the demand estimation, a brief review on retail demand is reviewed by Yao et al.(2003). Whitin (1955) is the reference work that combines pricing with inventory decisions. Mills (1959) refines Whitin's. Karlin and Carr (1962) present another demand model, the multiplicative form demand. Under this model the actual demand is given as the multiplication of the mean demand and the stochastic factor. Petruzzi and Dada (1999) offer an explanation as to why the pricing strategy appears to differ depending on how randomness is incorporated into the demand function. Subsequent works on the additive model include Ernest (1970), Young (1978), Lau

and Lau (1988) and Petruzzi and Dada (1999). The works on the multiplicative model include Nevins (1966), Zabel (1970), Young (1978) and Petruzzi and Dada (1999). There are economics papers on demand-price relationship as well but those papers related to our model is Federgruen and Heching (1997), and Petruzzi and Dada (2002). Petruzzi and Dada (2002) were a stochastic price dependent demand is proposed.

The interrelationship between price, capacity and demand are very few in literature. There is also a class of models that link quoted lead time, price and mean demand through a deterministic demand function. Dobson and Yano (2002) formulate an integer-programming problem which can be used to compute the best choices of product offering, prices, and MTS/MTO decisions simultaneously.

Even though a lot of work on optimal pricing can be found in literature , there are a very few researches on such a relationship between the price and available capacity in manufacturing area, the model Harris and Pinder(1995) proposed has the advantage of considering price versus a capacity choice.

In next section, we will review in detail the mathematical findings and inadequacy of available models to solve the price-capacity tradeoff formulation we have proposed. The new model will be explained in section 4.

3. Yield management in Manufacturing

Yield management (YM), or what is now called revenue management, employed within the airline industry involves a tiered pricing strategy with early discount pricing for price-sensitive customers. In the simplest case scenario involving only two fares, this means that the number of seats is offered for sale at a discount, while remaining are retained for sale to full-fare paying customers. The traditional YM goal is to define a decision rule for determining discounted versus full fare quantity, such that revenue/profit is maximized. This decision rule can be expressed as a simple function of the percentage difference in the fares (i.e. prices), and carefully defined probabilities associated with whether a potential buyer is likely to be a cost sensitive customer (i.e. CSC i.e.) or a Time Sensitive Customer (i.e. TSC) at particular points in time.

Normally, majority of the potential customers who book in advance are CSCs. As the time of delivery approaches and discount price items are

sold, the percentage of CSCs in the mix of customers decreases. On the other hand, TSC customers expect to have shorter "lead-time" and thus are less likely to be able to take advantage of discounted items relating to an advance purchase and are willing to pay full price.

The dynamics of demand in YM modeling is a major issue that makes information gathering and formulation of the demand patterns more attractive in many researches. From this point of view, the YM problem can be divided in to three main subjects as:

1-Sales patterns and market channels impact

2-Demand patterns by market segment

3- Effects of price changes on customer decisions

In manufacturing systems, the capacity definition varies according to the type of industry as well as the line of products. However, in general the capacity is a function of time usage of facility and equipment (or man-hour in manual technology industry). The capacity is measured in terms of production volume per hour, or per day, month or year. In particular, in auto industry the capacity is usually determined by the departments which are considered as the bottlenecked ones, such as body shop, paint shop and final assembly.

The designed capacity at each period is fixed and perishable, if not consumed within the period. This is due to high relationship between the availability of capacity in main manufacturing plant and spread supply chain. As the value chain is working in close cooperation to produce the final products. Each order can use the available capacity at any instant of time but preemption is not allowed. The main issue is how to selectively accept incoming orders so as to achieve a stated objective such as maximizing overall profit without being tardy on any accepted order and to manage seasonal demand to increase the utilization of the capacity in a robust and steady production Plan.

One of the distinct differences of car industry revenue management from its application to airlines is the degree to which price changes. Airlines typically have several discrete price classes and prices are fairly constant. There are often some restrictions to take advantage of the reduced prices such as advanced booking or Saturday night stay in an effort to segment business and leisure travelers. Airlines change prices by opening or closing these different fare classes. Car industry revenue management (RM) system also has discrete price classes for the

different types of delivery orders. We define R_n as the normal/highest price for the time sensitive customers (TSC), *i.e.* those who want to receive their good as soon as their order is registered. On the other hand, R_i is the discounted price for cost sensitive customers (CSC) who accept to receive the order later, at an agreed date which is suitable for the manufacturer in a reasonable duration, but in return in to pay a lower price.

The other difference between airline and manufacturing industry is the variable cost of producing a new product (*i.e.* passenger service in airline). In airline models the cost for serving a new customer is considerably low that can be neglected but in manufacturing sector the cost of materials is high and specially in car industry it goes up to 70-75% of the cost of vehicle. Fortunately, the capital intensiveness of most manufacturing plants as well as car industry, in special, makes such a capacity modeling useful. For the purpose of the model, let us define A as the variable cost of an average good being produced in manufacturing plant.

3.1. Demand function

The forecasted demand is the input to systems in revenue management. The range of forecasting methods applicable in RM vary from a simple times series and exponential smoothing models to complicated Bayesian forecasting techniques. The choice of the forecasting method varies from industry to industry, for different segments of the customers and for different sets of variables.

Mathematically, demand is a function of price. In literature, elasticity is a tool that is used to describe the relationship between two variables. It is defined as the percentage change in a dependent variable "caused" by a percentage change in an independent variable. The price elasticity of demand can be calculated at a specific price and quantity. This is called the point price elasticity and is differs at different prices. The price elasticity of demand can be formulated as:

$$\frac{dQ}{dR} \frac{R}{Q} = -b,$$

where the R is the market price or manufacturer revenue and Q is the demand. This may be rewritten as,

$$\frac{dQ}{Q} = \frac{(-b)dR}{R}.$$

Integrating both sides, results in $\ln(Q) = K - b \cdot \ln(R)$, where K is a constant of integration. Thus, this expression can be written as $Q = A \cdot R^{-b}$. Combining a price-demand function with the stochastic customer behavior resembles the actual demand function, we will discuss in section 4.

4. Capacity-price trade off model

In this section, we develop a model to determine the capacity and price or trade off between them. There are T time intervals, $0 \leq t \leq T$. By period t , it is meant the number of remaining periods is equal to t . Thus, reservation starts at period T , while the last period is denoted by $t=T$. At the beginning, the capacity of delivery of C is available and the objective is to determine the assigned capacity for each period, in order to maximize the total revenue.

The demand is stochastic and a function of price. In fact, demand follows a general function of $D_t = X_t R^{-b}$ where R is the price, b is the elasticity of demand and $X_t (t = 1, \dots, T)$ are independent identically distributed random variables. The definition of demand comes from the concept that there are two elements of uncertainty and price.

As mentioned before, there are two customer classes, time sensitive customers (TSC) and cost sensitive customers (CSC). TSC segment is obviously just one class while there are several cost sensitive customers, depending on the time of booking order. In each time interval, the capacity is offered to one class of customers only. Consider the following notation.

T : number of periods (or number of classes)

C : total capacity available at the beginning, i.e. at $t = T$.

C_t : planned capacity of period (or class) $t, t = 1, \dots, T$.

\hat{C}_t : remaining capacity available at the beginning of period $t, t = 1, \dots, T$.

TK : fixed cost of capacity

K : capital cost of one unit of capacity.

R_h : price at the highest level (normal price)

R_t : Price for class $t, t = 1, \dots, T$, where

$$R_T \leq R_{T-1} \leq \dots \leq R_2 \leq R_1 = R_h$$

A : variable cost of production

D_t : stochastic demand for class $t, t = 1, \dots, T$.

$$D_t = X_t R^{-b}$$

b : elasticity parameter of demand

X_t : stochastic demand element as a probability function

$F(D_t)$: cumulative distribution function of stochastic demand element for class $t, t = 1, \dots, T$.

Φ_t : total revenue for classes $t, t = 1, \dots, T$.

4.1. Revenue trade off for two classes

Let the available capacity be \hat{C}_t at the beginning of period $t, t = 1, \dots, T$, and the allocated capacity for this period is C_t , where clearly, $C_t \leq \hat{C}_t$. Then,

Revenue for class t

$$= (R_t - A)[E(D_t)] - KC_t \quad (1)$$

Where

$$E(D_t) = \int_0^{C_t} D_t dF(D_t) + C_t(1 - dF(C_t)) \quad (2)$$

The optimal capacity for each class can be found through partial derivative of revenue equation (1) with respect to D_t and then equating to zero.

$$\frac{\partial(\text{revenue})}{\partial C_t} = (R_t - A)(1 - F(C_t)) - K = 0$$

Then,

$$(R_t - A)(1 - F(C)) = K \quad (3)$$

Consider a special case of two-class optimization revenue management for a MTO policy. A general equality is found with a constant of K (cost of capital per unit capacity) as below:

$$(R_1 - A)(1 - F(C_1)) = K = (R_2 - A)(1 - F(C_2))$$

or

$$\frac{(R_1 - A)}{(R_2 - A)} = K = \frac{(1 - F(C_2))}{(1 - F(C_1))} \quad (4)$$

It can be understood from Equation (4) that the contribution from lost sales is equal to the cost of capacity. Also, the optimal ratio of R_1 to R_2 is equal to the ratio of lost sales probability of R_2 to R_1 in the simple YM problem.

4.2. Dynamic Programming Approach

Now we develop a dynamic programming approach to solve the model. To do that, each period (or each customer class) is considered as a stage of dynamic programming model and the remaining capacity at the beginning of the period (\hat{C}_t) as the state of the system. Let $\Phi_t(\hat{C}_t)$ represent the maximum of the expected total revenue of the periods t till the end, provided the remaining capacity is \hat{C}_t . Then, for period $t, t = 1, \dots, T$, the following recursive function can be developed.

$$\Phi_t(\hat{C}_t) = \text{Max}_R \{E(D_t)R_t + \Phi_{t-1}(\hat{C}_{t-1})\} - (A + K)\hat{C}_t \quad (5)$$

where,

$$\hat{C}_{t-1} = \hat{C}_t - D_t \quad (6)$$

Since $(A + K)\hat{C}_t$ is a constant value, we discard it from this point on. On the other hand, by substituting D_t with $X_t R_t^{-b}$ in (5) and defining $\phi_t = \Phi_t(\hat{C}_t) + (A + K)\hat{C}_t$ results in,

$$\Phi_t(\hat{C}_t) = \text{Max}_R \{E(X_t R_t^{-b})R_t + \Phi_{t-1}(\hat{C}_t - X_t R_t^{-b})\} \quad (7)$$

Lemma 1. Solving the recursive equations of (7) is equivalent to obtaining the set of $z_t^*, t = 1, \dots, T$, that optimizes the following set of equations.

$$r_t(z) = \frac{z - E[z - X_t] + r_{t-1}^* E[(z - X_t)^m]}{z^m} \quad (8)$$

where $m = 1 - \frac{1}{b}$ and

$$r_t^* = \text{Max}_z r_t(\hat{C}_t) = r_t^* \hat{C}_t^m \text{ and } r_0(z) = 0.$$

Furthermore, the optimal revenue function is as follows

$$\phi_t(\hat{C}_t) = r_t^* (\hat{C}_t)^m \quad (9)$$

Proof: The proof is by induction and we follow an approach introduced by Monahan, et al. [3]. Let define a new variable $z_t, t = 1, \dots, T$, as below.

$$z_t = \frac{\hat{C}_t}{R_t^{-b}} \quad (10)$$

Then,

$$\hat{C}_t - X_t R_t^{-b} = \hat{C}_t \left(\frac{z_t - X_t}{z_t} \right) \quad (11)$$

and,

$$(\hat{C}_t - E[\hat{C}_t - X_t R_t^{-b}])R_t = \hat{C}_t \left(\frac{z_t - E[z_t - X_t]}{z_t} \right) R_t \quad (12)$$

The recursive equation of (7) can be rewritten as follows.

$$\phi_t(\hat{C}_t) = \text{Max}_R \{(\hat{C}_t - E(\hat{C}_t - X_t R_t^{-b}))R_t + \phi_{t-1}(\hat{C}_t - (X_t R_t^{-b}))\} \quad (13)$$

We first prove (9) holds for $t=1$. Then, by considering $r_0(z) = 0$ and (11) as well as substituting R_t from (1), the recursive equation for the last period is as follows.

$$\begin{aligned} \Phi_1(\hat{C}_1) &= \text{Max}_R \{(\hat{C}_1 - E(\hat{C}_1 - X_1 R_1^{-b}))R_1\} \\ &= \text{Max}_R \{(Z - E[z - X]) \left(\frac{\hat{C}_1}{Z} \right)^m\} \end{aligned}$$

Since $r_1(z) = \frac{z - E[z - X_1]}{z^m}$ from (8), then,

$$\phi_1(\hat{C}_1) = \max_z \{r_1(z)(\hat{C}_1)^m\}$$

Now, let (9) holds for $(t-1)$, i.e. $\phi_{t+1}(\hat{C}_{t-1}) = r_{t+1}^* (\hat{C}_{t-1})^m$, then by considering (11), (12) and (8), the recursive equation of (7) can be rewritten as follows.

$$\phi_t(\hat{C}_t) = \max_z \{r_t(z)(\hat{C}_t)^m\} = r_t^* (\hat{C}_t)^m.$$

4.3. Determining z_t^*

The algorithm to solve the dynamic programming model actually consists of a two-step approach.

In the first stage, $z_t^*, t = 1, \dots, T$, are calculated by optimizing the set of equations (8). The important point about obtaining the optimal value for z_t^* is that the set of equations (8) are independent of capacity, \hat{C}_t . We first calculate z_1^* and then $z_1^*, z_1^*, \dots, z_T^*$.

In the second stage the optimal price or capacity is determined. Below, we explain the relation between price and capacity.

4.4. Relation between capacity and Price

After calculating the set of $z_t^*, t = 1, \dots, T$, as described above, one can determine either optimal price or optimal capacity, if the other one is given.

If the capacity is already established, the optimal prices for different classes are determined by the following relation, which is derived from (10).

$$R_t^* = \left(\frac{z_t^*}{\hat{C}_t}\right)^{1-m} \quad (14)$$

On the other hand, the objective can be establishing a capacity at the beginning. From (5), (7) and (9) the recursive equation for period $t, t = 1, \dots, T$, as follows.

$$\Phi_t(\hat{C}_t) = r_t^* \hat{C}_t^m - (A + K)\hat{C}_t + TK$$

Since the capacity is established for the first time, we consider a fixed cost of TK. Then, the optimum capacity for predefined demand characteristics is obtained by taking the derivative of $\Phi_T(\hat{C}_T)$ with respect to \hat{C}_T (by our notation $\hat{C}_T = C$) must be equal to zero. In that case,

$$C^* = \left(\frac{mr_T^*}{A}\right)^b \quad (15)$$

In case the capacity has to be established with unit revenue maximization, then the problem changes to the following mathematical model.

$$\text{Max}\Phi_{unit} = \frac{r_T^* C^m}{C} - \left(A + \frac{TK}{C}\right)$$

&

$$C^* = \left(\frac{(1-m)r_T^*}{TK}\right)^{1/m}$$

An optimal capacity allocation is the essential part of revenue management.

4.5. Dynamic Capacity-Price Solution Algorithm

From theoretical point of view a dynamic pricing problems deals with environments where demand is random and supply is fixed and determined. In contrary, a revenue management problem solves the problems while demand is random and price is fixed and determined. To this point, we have introduced a general formula for capacity- price tradeoff for each class (i.e. $(z_t^*, t = 1, \dots, T)$) which acts as a linkage between two disciplines. Now we introduce an extended relationship between demand and price, which make it possible to find the optimal capacity and prices for all classes, simultaneously.

Refer to equation (2), Revenue for each class (call it each period) can be written as :

$$\theta_t = R_t \left[\int_0^{q_i} D_t dF(D_t) + \int_{q_i}^{\infty} C_t dF(D_t) \right] - (K' + A) C_t \quad (16)$$

Where

θ_t : Revenue for class $t, t=1, \dots, T$

K' : the capital cost of unit capacity

q_i : Planned capacity of class (i.e. C_t) ;used as notation for the variable with content of C_t

Introducing demand function as $D_t = X_t R^{-b}$, substitution in (16) and replacing $(K' + A)$ with K , we will have the following general tradeoff function for each period

$$\theta_t = R \left[\int_0^{q_i} D_t dF(D_t) + \int_{q_i}^{\infty} C_t dF(D_t) \right] - K C_t \quad (17)$$

Since $K C_t$ is a constant value, we will discard it from this point forward. Now, Let $\theta_t(C_t)$ represent the total expected revenue of period t until the remaining with a known capacity of C_t . Then for period $t, t = 1, \dots, T$, the following recursive equation can be developed.

$$\theta_t(C_t) = \text{Max}_{q_i \leq C_t - \sum_{i=1}^{t-1} c_i} \{R_t [\int_0^{q_i} D_t dF(D_t) + \int_{q_i}^{\infty} C_t dF(D_t)] + \theta_{t-1}(\hat{C}_t - C_t)\} \quad (18)$$

Proposition 1: let $D_t = X_t R^{-b}$ and let the revenue – lost sales trade off formulated as equation (18) and a continues-probability distribution function for demand, the optimal capacity allocated for each period will be found through a dynamic capacity reallocation with the following relationship.

$$\theta_t(C_t) = \text{Max}_{C_t \leq \hat{C}_t = C_t - \sum_{i=1}^{t-1} c_i} \{(z_t^{1-m} c_t^{m-1} [\int_0^{c_t} x z_t^{1-m} c_t^{m-1} f_x(x) dx + \int_{c_t}^{\infty} c_t f_x(x) dx]) + \theta_{t-1}(\hat{C}_t - C_t)\} \quad (19)$$

Proof: the proof follows by induction of demand function in equation (18). Assuming that demand function has a probability density function of $f_x(x)$, The equation will transform to following relationship:

$$\theta_t(R_t, C_t) = \text{Max}_{C_t \leq \hat{C}_t = C_t - \sum_{i=1}^{t-1} c_i} \{(R_t^{-b} [\int_0^{c_t} x R_t^{-b} f_x(x) dx + \int_{c_t}^{\infty} c_t f_x(x) dx]) + \theta_{t-1}(\hat{C}_t - C_t)\} \quad (20)$$

This is due to probability theory, since $D=X.R^{-b}$ then $P(X < x) = P(D < R^{-b}x)$. Taking the derivative of both sides with respect to x gives

$$f_D(D) = f_D(R^{-b}x) = \frac{f_X(x)}{R^{-b}} = \frac{1}{R^{-b}} f_X\left(\frac{D}{R^{-b}}\right) \quad \& \quad \partial D = R^{-b} \partial x \quad (21)$$

We have shown that the optimal relation between capacity availability and price (z values) can be found through dynamic programming (equation 13). Then by considering (21) and Z_t values and substitution in (20) results into a new recursive relationship with one variable (i.e. capacity) as stated in (19) above .

Solving recursive equations as described in (19), the optimum capacity and relative prices planned for each class will be calculated. □

Figure 2, summarize the Solution procedure for dynamic pricing and capacity allocation we have developed.

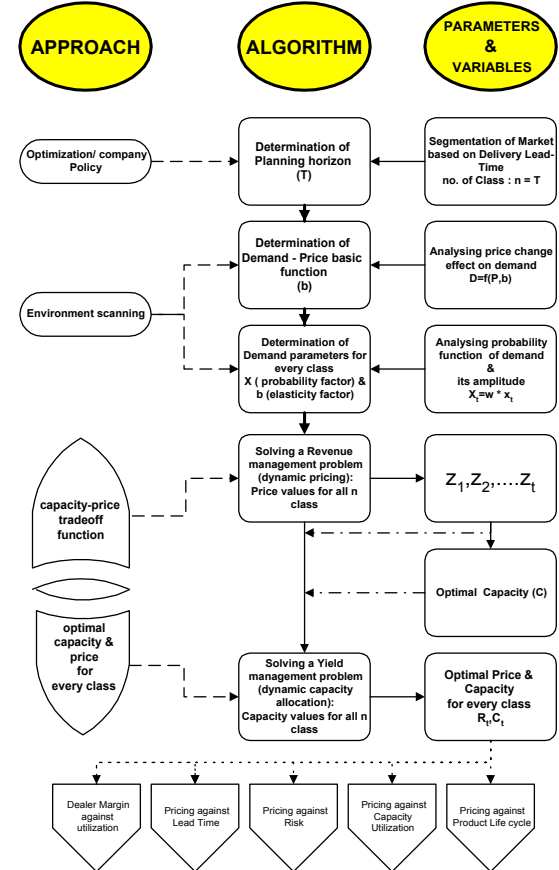


Figure 2- Flow chart for Manufacturing YM mathematical model

The approach starts with company & market assumptions and combines in a capacity-price trade off solution method. General algorithm processes are shown in middle & related parameters and variables in right.

5. Extended framework for optimal Solution

So far, we have developed a general relationship between optimal capacity for multi class customers and related price offer while maximizing revenue for the total problem. In this section, we will discuss some managerial and computational implication of the model due to changes in demand and price policy settings.

What is the effect of a fixed multiplier on demand function on the optimal solution?

If the prices over time can be modeled as $R_t = \alpha R_{t-1}$. what would be the relationship between capacities allocated and price changes multiplier on an optimal scenario?

As the risk of demand which means the probability of discontinuing demand increases, what happens to optimal solution?

5.1. Pricing against uniform demand shift

There are cases where the demand behavior on certain series of time is similar in nature but the difference is the amplitude of demand. We will prove the following proposition to reduce calculation effort.

Proposition 2: if demand changes according to a multiplier (i.e. $newX_t = nX_t$ or $D_t = nX_t R^{-b}$), then optimal capacity will change in proportion to multiplier under the condition of fix prices.

Proof:

From Monahan (2004), it is proved that for a $newX_t = nX_t$, there is a relationship of $new(\hat{Z}_t) = n\hat{Z}_t$. Then from probability theory, we have:

$$P\{nX < x\} = P\{D < nxR^{-b}\} \quad \text{or}$$

$$\int_{-\infty}^{x/n} f_x(x)dx = \int_{-\infty}^{nxR^{-b}} f_D(D)dD$$

by derivation from both sides , we have :

$$\frac{1}{n} f_x(x) = nR^{-b} f_D(D) . \text{ then}$$

$$f_D(D) = \frac{1}{n^2 R^{-b}} f_x(x) \quad \& \quad dD = nR_t^{-b} dx \quad (22)$$

Let's consider $D_t = nX_t R^{-b}$ and (22) and substitute in equation 19, then equation (19) will change as follows:

$$\theta_t(R_t, C_t) = \text{Max}_{C_t \leq \hat{C}_t = C - \sum_{i=t+1}^T c_i} \{ (R_t^{-b} [\int_0^{c_t} nx.R_t^{-b} \frac{1}{n^2 R_t^{-b}} f_x(x) nR_t^{-b} dx + \int_{c_t}^{\infty} \frac{1}{n^2 R^{-b}} f_x(x) nR_t^{-b} dx]) + \theta_{t-1}(\hat{C}_t - C_t) \} \quad (23)$$

Now, let us assume that a proportional change in capacity elements will happen or $new.C_t = nC_t$ and by substitution, we have:

$$\theta_t(nC_t) = \text{Max}_{C_t \leq \hat{C}_t = C - \sum_{i=t+1}^T c_i} \{ (R_t^{-b} [\int_0^{nc_t} x.(nz)_t^{1-m} .(nc)_t^{m-1} f_x(x).dx + \int_{nc_t}^{\infty} (nc_t) f_x(x).dx]) + \theta_{t-1}(n\hat{C}_t - nC_t) \} \quad (24)$$

let us define new variables as $\hat{\zeta}_t = n\hat{Z}_t$ &

$\gamma_t = nC_t$ and $\hat{\gamma}_t = n\hat{C}_t$ substitute in (24) , then

$$\theta_t(\gamma_t) = \text{Max}_{\gamma_t \leq \hat{\gamma}_t = \gamma - \sum_{i=t+1}^T \gamma_i} \{ (R_t^{-b} [\int_0^{\gamma_t} x.(\hat{\zeta})_t^{1-m} .(\gamma)_t^{m-1} f_x(x).dx + \int_{\gamma_t}^{\infty} \gamma_t f_x(x).dx]) + \theta_{t-1}(\hat{\gamma}_t - \gamma_t) \} \quad (25)$$

Comparing (25) with (19) will prove; $new(\hat{C}_t) = n\hat{C}_t = \gamma_t$.

The above-mentioned relationship shows that in optimal adjustment of capacity, on conditions of demand changes by a multiplier of n, the prices will not change if the total capacity multiplied by a coefficient of n. □

5.2. Pricing against Lead –Time

As the customer willingness to advance purchase of capacity is related to increasing risk of usage for the customer, the demand forecast can be modeled as if cumulative hazard rate for demand increases in accordance to lead-time. There exists a reverse relationship between optimal prices and lead-time. A usual practice complies with the mathematical model for demand behavior.

Proposition 3: for cases where the cumulative hazard rate for demand classes has the general increasing trend of $\Lambda h_t \geq \Lambda h_{t-1}$ over time, prices on optimal assignment increases over time or $R_t \geq R_{t-1}$.

Proof: consider $\Lambda h_1 \leq \Lambda h_2 \leq \Lambda h_3, \dots \leq \Lambda h_t$ as the cumulative hazard rate for demand function over time. From definition of cumulative hazard rate, we have:

$$\Lambda h_t = \int_{i=0}^{i=t} h_i(D).dD \quad \text{where}$$

$$h_t(D) = \frac{f_t(D)}{S_t(D)} \quad \text{and,}$$

$$S_t(D) = 1 - F_t(D)$$

Also from definition of CDF function, it can be shown that:

$$f(D) = \frac{d}{dD} F(D) \text{ or } f(D) = -\frac{d}{dD} S(D)$$

Therefore

$$h_t(D) = \frac{f_t(D)}{S_t(D)} = -\frac{\partial \ln(S_t(D))}{\partial D} \quad \&$$

$$\Lambda h_t(D) = -\ln(S_t(D)) \quad (26)$$

Combining assumption of the proposition, and equation (26) we have:

$$\ln(S_1(D)) \geq \dots \geq \ln(S_{t-1}(D)) \geq \ln(S_t(D)) \quad (27)$$

Then applying equation (4) we have:

$$\frac{R_t}{R_{t-1}} = \frac{1 - F_{t-1}(\hat{C}_{t-1})}{1 - F_t(\hat{C}_t)} = \frac{S_{t-1}(\hat{C}_{t-1})}{S_t(\hat{C}_t)} \quad (28)$$

Considering (27) & (28) then

$$\therefore R_t \geq R_{t-1} \quad \square$$

In practice, there are situations where the shortcoming of capacity causes to make delay for delivery or in other words make the lead-time longer than expected. For these situations, the customers usually expect for discount on longer lead times. The following proposition shows how to deal with such environments through redefinition of price elasticity.

Lemma 2: for cases where Lead time effect on demand has to be enforced and demand parameter for elasticity has a general formula of $D_t = nX_t R^{-g(t)}$, price will change through a scale down factor of $g(t)/b$.

Proof: considering (14) and by dividing Z values of any class (i.e. period t) to final class (i.e. period T), we have :

$$\frac{R_T}{R_t} = \left(\frac{Z_T}{\hat{C}_T} \right)^{1-m} \bigg/ \left(\frac{Z_t}{\hat{C}_t} \right)^{1-m}$$

Also, consider that the availability of capacity doesn't change due to not receiving any order during consecutive periods (i.e. $\hat{C}_T = \hat{C}_t$).

Then,

$$R_T = R_t \left(\frac{Z_T}{\hat{C}_T} \right)^{1-m} \bigg/ \left(\frac{Z_t}{\hat{C}_t} \right)^{1-m} = R_t \left(\frac{Z_T}{Z_t} \right)^{1-m} \quad (29)$$

Considering (29) & demand function stated in lemma and solving simultaneous equation, we have:

$$R_t = R_T * Z_t^{1-m} * Z_T^{m-1} \quad \&$$

$$g(t) = \frac{\text{Log}(R_T)}{\text{Log}(R_T * Z_t^{1-m} * Z_T^{m-1})} \quad (30)$$

Now, let us define a new variable for price at period t with following definition,

$$R' = R^{g(t)/b} \quad (31)$$

Where b is a scale factor and g(t) is a function of time representing lead time factor.

Considering (30) and (31), the demand function will change to a new function of $D_t = nX_t (R')^{-b}$.

It is obvious that the by introduction of new variable the problem will changes to its original form and can be solved through a dynamic programming approach using equation (19). The result prices of dynamic programming model are due to changes by substitution of R' with $R^{g(t)}$. □

It is interesting for reader to note that by introduction of this lemma, lead-time effect on a stable market can be investigated through a reverse analysis of actual data with theoretical relationship.

5.3. Pricing against capacity utilization and market segments

Proposition 4: for customers where the opportunity cost is more attractive than lead time, there exist an upper bound of discount to sell unsatisfied capacity in order to minimize revenue reduction which is the most beneficial for both buyer and seller.

Proof: consider demand in period t-1 is less than the capacity assigned that means $D_{t-1} \leq C_{t-1}$. Let us define the demand shortcoming as \mathcal{E} , then from (10) reduction in price for next coming segment is expected to be as following:

$$\frac{R_t^{new}}{R_t^{Exp}} = \frac{\left(\frac{Z_t}{C_t + \varepsilon}\right)^{1-m}}{\left(\frac{Z_t}{C_t}\right)^{1-m}} \Rightarrow$$

$$R_t^{new} = R_t^{Exp} \frac{\left(\frac{Z_t}{C_t + \varepsilon}\right)^{1-m}}{\left(\frac{Z_t}{C_t}\right)^{1-m}} = R_t^{Exp} \left(1 - \frac{\varepsilon}{C_t + \varepsilon}\right)^{1-m}$$

Therefore, lower bound discount for unfulfilled capacity is equal to total expected discount for next coming class to demand shortcoming of ε for previous segment or in mathematical terms as:

Discount _per _unit _capacity

$$\leq \left(1 - \frac{C_t R_t^{Exp} \left(1 - \left(1 - \frac{\varepsilon}{C_t + \varepsilon}\right)^{1-m}\right)}{\varepsilon * R_{t-1}}\right) * 100 \quad (32)$$

Figure (3) shows the behavior of such multiplier where discount will increase by coefficient of "shortage to capacity ratio" on a parabolic curve. In addition, discount will increase as elasticity of demand increases which is consistent with customer behavior.

Such a discount calculated as (32) may apply into agreements with wholesale customers. These agreements are a trade off risk for wholesale customer to have capacity on lower cost versus of lost sales due to non-fulfilled demand for seller.

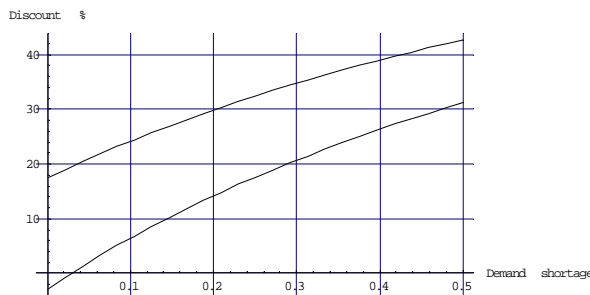


Figure3 - Price Multiplier and discount behavior

5.4. Pricing against product life cycle and company image

There are policies for management to avoid price discounts. Therefore, such policies should be taken care in manufacturing industry to prevent company image distortion. Also, there are certain cases where, there is a recommended policy from management to minimize number of price changes over classes(over time) in order to maintain the stability of company image.

Lemma 3 : *if management decides to restrict relative prices to consecutive classes over time by a coefficient (i.e. α), there exists a relationship between coefficients for optimal prices versus optimal capacity allocated which is consistent with elasticity of demand (1-m).*

Proof: with refer to equations (12) the following relationship exists between consecutive optimal variables.

Revenue for a period:

$$E(\phi_t) = \hat{C}_t - E[\hat{C}_t - X_t(\alpha' p_i^{-b})] = \hat{C}_t \left(\frac{z_t - [z_t - X_t]}{z_t}\right)$$

Let's consider (10) and applying for consecutive Z_t 's as follows:

$$\frac{Z_t}{Z_{t-1}} = \frac{\hat{C}_t}{\hat{C}_{t-1}} * \left(\frac{R_t}{R_{t-1}}\right)^b$$

Now let's define $\alpha_t = \left(\frac{R_{t-1}}{R_t}\right)$ then ,

$$\alpha_t = \left(\frac{Z_{t-1}}{Z_t}\right)^{1-m} \left(\frac{\hat{C}_t}{\hat{C}_{t-1}}\right)^{1-m}$$

where $m = 1 - 1/b$

or

$$\alpha_t = \gamma_t^{1-m} \left(\frac{\hat{C}_t}{\hat{C}_{t-1}}\right)^{1-m} \quad (33)$$

Where $\gamma_t = \left(\frac{Z_{t-1}}{Z_t}\right)$ and $m = 1 - 1/b$

The above-mentioned relationship (i.e. equation (33)) will help any devised search algorithms to find optimal solution. In addition, when management sets the policies for consecutive

prices, the capacities for each class of customers are already setup in an optimal relationship.

□

5.5. Optimal solution for Price step down scenario

Now, let us consider that a problem is solved by setting $\alpha = \alpha_0$ according to equation (24), and resulting revenue is ϕ_0 . Changing $\alpha = \alpha_0 + \varepsilon$ will change the revenue function as $\phi = \phi_0 \pm \varphi$. Then, Solving for all α , we can find optimum price step down coefficient.

Lemma 4: *There exists a lower and upper boundary for α that bounds α feasible selection space.*

Proof : Solving a dynamic yield-manufacturing problem using equation (19), there exist maximum and minimum prices over classes. Thus Minimum amount for α would be:

$$\alpha \geq \left(\frac{R_0}{R_T}\right)^{1/T}$$

From proposition 4 and for real life environment depicted in proposition 3, then we have $R_{t-1} \leq R_t \dots \leq R_T$, and therefore $\alpha_t \leq 1$, thus

$$\alpha \geq \left(\frac{R_0}{R_T}\right)^{1/T} \quad \& \quad \alpha_t \leq 1$$

$$\Rightarrow \left(\frac{R_0}{R_T}\right)^{1/T} \leq \alpha \leq 1$$

Now, let us assume -as it is normal in real practice that there is a constraint of minimum price (i.e. L). Then from definitions, there exist a boundary for optimal price coefficient as below:

$$\alpha \leq 1 \quad \& \quad \alpha^T \geq L \quad \Rightarrow L^{1/T} \leq \alpha \leq 1$$

Combining above relations, we have:

$$\Rightarrow \text{Max}\left\{\left(\frac{R_0}{R_T}\right)^{1/T}, L^{1/T}\right\} \leq \alpha \leq 1 \quad (34)$$

□

Figure (4) shows the feasible selection space before and after application of equation (34). It is obvious that such a relation will decrease the size of problem.

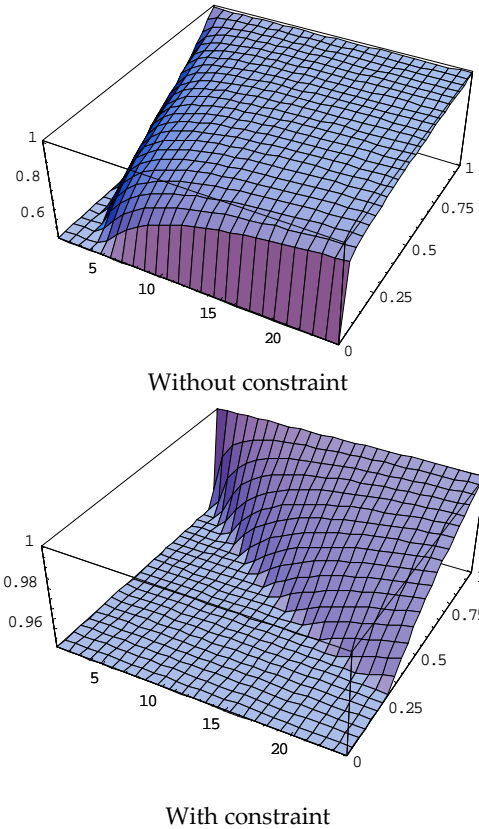


Figure 4- Efficiency of search and feasible solution space

5.6. Dealer margin against capacity utilization

Dealers in a advanced selling approach have a major effect to convince customers to wait and take the benefit of discounts. Therefore, A policy should be selected to take care their interest as well as sellers benefit.

Proposition 5: *for dealers who purchase the capacity in advance to meet their customer demand, there exists a price – sales policy criteria that motivate advanced selling of unsatisfied capacity in order to minimize reduction of revenue for manufacturer and to increase revenue of dealer.*

Proof: consider there exist a known hazard rate for demand continuity on period t as h(t). From definition of statistics the probability of purchasing for customers with booked advanced order on their assigned lead time is equal to survival function of demand or

$$S(t) = \int_0^t f(x)dx$$

case I :where the dealer has purchased the capacity by himself to take the benefit of sales and take the risk himself.

As the excess capacity due to dispensed orders has to be sold on delivery date , therefore following cost and benefit relations exist:

Lost of sales due to q excess capacity:

$$\int_{C+q}^{\infty} (R_t * w)(D - (C + q))dF(D)$$

Revenue of sales due to q excess capacity:

$$\int_{-\infty}^{C+q} (R_T - R_t)((C + q) - D)dF(D)$$

Where

$q = Q_t(1 - S(T - t))$:dispensed capacity orders

$T-t$: The time difference between advance order and time of sales.

w : Cost of unsold order as a percentage of capacity expected revenue(i.e. cost of investment)

Q_t : Total capacity booked on period t by dealer

Thus from cost-benefit tradeoff, and taking the derivative of both sentences to $(C+q)$ and letting it to zero , we have:

$$(R_T - R_t)F(C + q)^* \geq (R_t * w)(1 - F(C + q)^*)$$

or

$$\frac{(R_T - R_t)}{(R_t * w)} \geq \frac{(1 - F(C + q)^*)}{F(C + q)^*} \rightarrow$$

$$\frac{R_T}{R_t} \geq 1 + w \frac{1 - F(C + q)^*}{F(C + q)^*} \quad (35)$$

The above relation shows in case the lead time is long , the probability of high demand (or shortage of capacity)has to be considered so as to convince dealer to take care of capacity advance selling scenario.

This means that the seller has to move more to advance sales in order to decrease available capacity on delivery date to recover dispensed orders or in mathematical view:

$$C \rightarrow 0 \ \& \ R_T \rightarrow \text{high_end}$$

case II :where the dealer has the role of just a dealer service and sales agent that take the benefit of margin for every sales he has made.

Let us assume that total risk for dispensing of orders is cumulative hazard rate of demand which is equal to

$$\Lambda(t) = \int_0^t h(x)dx$$

Considering the margin for dealer as M , the total lost revenue due to dispensing of orders is:

$$L_i = V * \Lambda(t) * Q_i \quad \&$$

$$L_i^{unit} = V * \Lambda(t) \quad \& \ q = Q * \Lambda(t)$$

Where

L_i : Lost revenue for dealers advanced sales in period i with lead time of t

L_i^{unit} : Unit lost revenue for dealers advanced sales in period i

V : dealer margin for sales of one unit of capacity

Q_i : advanced order for period i

q : risky orders quantity

$\Lambda(t)$: Cumulative hazard rate for lead time of t

As explained before, the following cost and benefit relations exist:

Lost of sales due to q excess capacity:

$$\int_{C+q}^{\infty} v * (D - (C + q))dF(D)$$

Revenue of sales due to q excess capacity:

$$\int_C^{C+q} R_T ((C + q) - D)dF(D)$$

$$v(1 - F(C + q)) \leq R_T (F(C + q) - F(C))$$

or

$$v \leq \frac{R_T (F(C + q) - F(C))}{(1 - F(C + q))} \ \& \ v \leq V$$

or

$$v = \text{Min} \left\{ \frac{R_T (F(C + q) - F(C))}{(1 - F(C + q))}, V \right\} \quad (36)$$

the amount paid to dealer for recovery cost of dispensed customer is less than usual promotion cost and also the margin gained from excess revenue due to increase in availability of capacity.

Through extension of findings from base Yield management theory in manufacturing, has reached to a series of conclusion we have proposed. The following flow chart shows the general framework for manufacturing yield management. The process includes outcomes of

the model for optimal target setting for pricings by manufacturer.

6. Numerical examples

From definition, Price elasticity, the percentage change in demand per percentage change in price describes the sensitivity of sales volume to price changes. In Figure (5), demand is relatively inelastic over price changes from R₂ to R₃ as very little additional sales result (ΔQ₂ is small), while very elastic from R₁ to R₂ (ΔQ₁ is quite large). Over the range P₁ to P₂ the price has crossed a competitor' price PC, generating the volume increase (decrease) whereas from R₂ to R₃ it is still priced between two competitors, or already has the lowest price.

Figure (5) is a simplified illustration of price elasticity within Iran car industry. This characteristic can be observed through general mathematical economics that we are not going to discuss it in detail. We put such a demand observation and historical data in the general demand model. It is clear that the possibility of using $D=K.R^{-b}$ as demand function has been proved. As it is shown in figure, for Iran car industry (as well as all over the world) demand changes are due to negative price effect as well as proposed demand function

$$\left(\frac{\Delta D}{\Delta R} = -b(R^{-b-1}) \leq 0.\right)$$

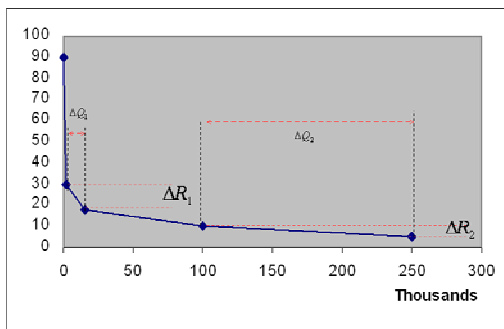


Figure 5 – Price elasticity factor in IRAN auto industry

Illustration I, demand function estimation:

Consider a demand function with K as a constant parameter and solving a simultaneous equation for a price and its related demand, we will have the following relationship.

$$b = \frac{\text{Log}(D_1) - \text{Log}(D_2)}{-\text{Log}(R_1) + \text{Log}(R_2)} \quad \&$$

$$K = e^{\frac{\text{Log}(D_2)\text{Log}(R_1) - \text{Log}(D_1)\text{Log}(R_2)}{\text{Log}(R_1) - \text{Log}(R_2)}}$$

Where b is the elasticity of demand, R is price and D refers to demand volume.

Taking in to account market data for B⁺ and C segment car and solving simultaneous equations of two segments through mathematica software will estimate parameters of the Demand function as below: (figure 6)

$$\{ \{b \rightarrow 1.28933, K \rightarrow 1.95448 \times 10^6\} \}$$

$$f = (1.95448 * 10^6) * R^{-1.289331053138751}$$

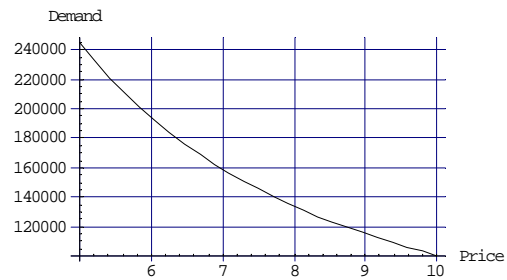


Figure 6– Price elasticity factor in IRAN auto industry

Illustration II, dynamic Manufacturing Yield Plan:

Consider a four period DMYP for a car manufacturer with capacity available of 62500 each period and demand elasticity b of 1.289. Also, assume that the demand probability function for each period is as following:

Planning Period	Sales period	Probability function	parameters
T	4	Uniform	b=1.2, a=0
T-1	3	Uniform	b=.8, a=0
T-2	2	Uniform	b=.45, a=0
T-3	1	Uniform	b=.2, a=0

TABLE 1 - SAMPLE PROBLEM DEFINITION

Solution:

Stage 1: let's calculate the Z functions for the four periods by introducing related formula. Results of above calculations are summarized in following table:

Planning Period	R values	Z values	Parameters	Iteration
T	1.20168	1.56943	b=1.2, a=0	4
T-1	0.743753	0.912243	b=.8, a=0	3
T-2	0.459691	0.390118	b=.45, a=0	2
T-3	0.147861	0.174749	b=.2, a=0	1

TABLE 2 - SUMMARY OF FINDINGS

Stage 2: Calculation of Price and capacity related quantity and solving a dynamic programming problem would result to following capacity allocation.

It can be easily verified that Solving for the required capacity in the model through dynamic programming approach yields to following result. The result of allocation to four class customers will be as follows:

	T	T-1	T-2	T-3
Capacity	27500	20000	10000	5000
(C)				
Price (R)	13.2	5.6	3.3	1.27

TABLE 3 - SAMPLE RESULT FOR THE PROBLEM

It is interesting to check the affect of demand changes on the price and allocated capacity to each class that is in line with expectation from the model behavior.

The following table shows the effect of demand changes on price and capacity for scenario's with uniform demand behavior but with different mean.

		T	T-1	T-2	T-3
Scenario II	Probability	(0,1.2)	(0,1.2)	(0,1.2)	(0,1.2)
	Z values	2.51951	1.94877	1.42068	1.04849
	r values	1.89104	1.4959	1.06995	.059366
	Capacity(C)	15000	15000	15000	17500
	Price (R)	30.	14.6	8	5.1
Scenario III	Probability	(0,0.2)	(0,0.2)	(0,0.2)	(0,0.2)
	Z values	1.25292	.324794	.23678	.174749
	r values	.881584	.372577	.2666488	.147861
	Capacity(C)	10000	15000	17500	20000
	Price (R)	24.332	4.19	2.17	1.27
Scenario I (Original)	Probability	(0,1.2)	(0,.8)	(0,.45)	(0,.2)
	Z values	1.56943	0.912243	0.390118	0.174749
	r values	1.20168	0.743753	0.459691	0.147861
	Capacity(C)	27500	20000	10000	5000
	Price (R)	13.2	5.6	3.3	1.27

TABLE 4 - COMPARISON SHEET FOR DIFFERENT MARKET BEHAVIOR

As it is clear from the table, with increasing chance for demand, the prices are higher and capacity allocation goes for periods with higher probability of demand (scenario I). In addition, the prices for higher demand environment (Scenario II) are always higher than other scenarios. Scenario III shows another important

phenomenon by allocating more capacity for advanced sales in order to decrease the cost of over demand capacity providing that capacity are kept the same in all scenario's.

Also it is obvious from the table that changes in price are relatively high, where in usual practice such a change have to be managed. The important point is to apply "lemma1" in order to manage such deviation. Another solution is to apply Lemma 3 to smooth the prices.

7. Conclusions and Further research

The major concern for all practitioners is how to utilize the capacities in favor of all stockholders, including customers and shareholders. This concept has been forgotten in most planning process. We argue that in most cases the objective is just to optimize the shareholders value in short terms, which is just to optimize price or capacity allocation. The concept, we have put in practice is to lead the market by utilization of resources to get the most benefit for the shareholders in long run while keeping the productivity and resource utilization as competitive as its rivals through a simultaneous price-capacity trade off model.

The model provided can be used in a wide area of products and manufacturing systems with Assembly to order and make to order systems. The approach utilizes a dynamic programming algorithm to cope with uncertainties in market as well as the capacity availability in favor of utilization and strategic market leadership plan.

The basic concept of our model has implications on the management policies from the traditional make to stock attitude to make to stock end even further to capacity sales. This concept will be brought into practice with global competition effect on market and increasing customer oriented products where the products has to meet individual requirements. The propositions provided in paper covers the basic needs for policy setting for an interrelated discipline of marketing and capacity planning which is not touched with current marketing theories or planning optimization algorithms.

The solution procedure devised for the problem definition suggest a two step dynamic programming in which the interface between the steps are independent from management decision making procedure. This means that the first step will provide a market behavior indication, which can be provided in a handbook for practitioners' usage. This can be a further

research area to search and categorize the most useful demand patterns and its related market factors (i.e. Z values).

Another area of research is the extension of principle provided to lean enterprises where the value chain integration on price – capacity trade-off problems is the major issue for further development of the model. In our proposed approach, we could address the market behavior in favor of capacity utilization within a network of supply. Through this way of thinking, it brings a new concept for rivalry of networks instead of front-end manufacturer competition, which is the current practice of global market players. As argued by Michael porter, supply network has the major role for core competency of a market leader. Therefore, as there are not proper feedback systems to revise the demand parameters for a network in such a way that could convince the partners even within our problem definition, this is perhaps the logical next step in further research.

An approach in which the size of the planning periods and partners increases during the planning horizon can easily be adapted by further research proposed in general in this paper. In fact the only implication is that the planning horizon must be extended, and redefined. When, for example, we would consider periods of a day in the first week of the planning horizon, and periods of a week in the subsequent 10 weeks, we could redefine the planning horizon ($t = 0, \dots, 10$ weeks) as follows: $t = 0, \dots, 6$ days, $t = 7, \dots, 16$ weeks. An important implication of this extension is that the size of the entire model increases, and may become harder to solve. Whether this extension leads to a higher utilization of resources and market leadership is subject of further research.

We have shown the effects of a single parameter changes on capacity planning and pricing which help the decision makers to investigate the policies. Based on some decision criteria, such as costs or profit, aggressive or defensive competition policies, the pricing will be affected. Effects of Such a strategic policy on capacity planning convey new ideas on strategic capacity management and pricing which is another area of research for bench marking.

In brief, we studied capacity-planning problems in a make-to-order manufacturing environment in this paper. We argued that many manufacturing companies that even produce semi standard items will confront with the concept of pull system to avoid unreasonable

costs of market deviations. Therefore, such a planning process can be modeled even for a daily usage of planners in plants to bring the capacity utilization and pricing trade off into practice. This situation can be a further research area to generate a trade off curves for most popular market behaviors in a handbook or software tool for middle managers and supervisors.

The challenging paradigm shift for decision makers to apply the model is to adjust dynamically their manufacturing system according to market behavior that is consistent with model dynamics presented in paper.

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